

Traveling-Wave Couplers for Longitudinal Beam-Type Amplifiers*

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Summary—The equations governing traveling-wave interaction between an electron beam and a slow-wave circuit are formulated in terms of amplitudes of circuit mode and slow and fast space charge modes. The resulting equations are solved to find expressions for the matrix which relates the mode amplitudes at the output of the traveling-wave coupler to the mode amplitudes at the input. The properties of this matrix are discussed and numerical values given for Kompfner Dip.

Matrices for velocity jumps and drift regions are also given, and the characteristics of couplers which are preceded by or followed by a drift region and velocity jump are discussed.

It is shown that necessary and sufficient conditions for the removal of beam noise from the fast space-charge wave by any lossless coupler are that, for a circuit input, there be no circuit output ($M_{11}=0$) and no slow space-charge wave output ($M_{21}=0$).

These results are then applied to the design of fast space-charge wave couplers for longitudinal beam type parametric amplifiers.

I. INTRODUCTION

THE prospect of obtaining a very significant decrease in the noise figure of electron beam type microwave amplifiers through the use of the parametric principle has stimulated considerable work on beam-type parametric amplifiers. Conventional longitudinal beam amplifiers depend critically on the negative power flow associated with the slow space-charge wave, whereas parametric amplifiers can be made to use the fast space-charge wave which has positive power flow. The significant distinction to be noted here is that noise can be completely removed from the fast space-charge wave whereas noise on the slow space-charge wave cannot. Parametric amplification in electron beams has already been analyzed and discussed by Louisell and Quate,¹ and the purpose of this paper is to describe the properties of a certain class of couplers which make it possible to couple to the fast space-charge wave only. While it is also possible to construct fast-wave couplers using resonant cavities, the possibility of using a traveling-wave interaction immediately suggests itself as a method with potentially greater bandwidth.

The simplest coupler of this type, conceptually, is a large QC traveling-wave structure operated at the Kompfner Dip.² When QC is large, coupling to the slow

space-charge wave is negligible and an almost complete interchange of energy takes place between the circuit and the fast space-charge wave.³ Any circuit input is transferred almost completely to the fast space-charge wave and any disturbance on the fast space-charge wave is transferred almost completely to the circuit. Any noise or signal on the slow space-charge wave passes through the interaction region unchanged. Used as an input coupler, noise would be completely removed from the fast space-charge wave. As an output coupler it would not be sensitive to the noise which remains on the slow space-charge wave. Such a coupler is very attractive, and it is of interest to inquire whether or not the large QC restriction is essential.⁴

We expect that as QC is decreased, coupling to the slow space-charge wave becomes important and that the noise may no longer be completely removed from the fast space-charge wave. The following analysis was performed in an attempt to answer the following types of questions about operation at small and intermediate values of QC :

- 1) How large is the coupling to the slow space-charge wave?
- 2) How much noise remains on the fast space-charge wave after passing through such a coupler?
- 3) Does there exist a set of values of b and CN for which there is no coupling to the slow space-charge wave?
- 4) Is it possible, with the aid of velocity jumps and drift regions, to achieve coupling to the fast wave only and removal of the beam noise from the fast space-charge wave?⁵

Since these questions are couched in terms of mode amplitudes it was found to be convenient to first formulate the equations of traveling-wave interaction in these terms by introducing a transformation from the physical variables: circuit voltage, beam current, and beam velocity to the three mode amplitudes. The result is a derivation of the coupled mode equations.^{3,6} When for-

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¹ W. H. Louisell and C. F. Quate, "Parametric amplification of space-charge waves," *Proc. IRE*, vol. 46, pp. 707-716; April, 1958.

² R. Kompfner, "On the operation of the traveling wave tube at low level," *Brit. J. IRE*, vol. 10, pp. 283-289; August-September, 1955. See also H. R. Johnson, "Kompfner Dip Conditions," *Proc. IRE*, vol. 43, p. 874; July, 1955.

³ R. W. Gould, "A coupled mode description of the backward wave oscillator and the Kompfner Dip condition," *IRE TRANS. ON ELECTRON DEVICES*, vol. ED-2, pp. 37-42; October, 1955.

⁴ Large QC may be obtained by reducing the helix to beam coupling thus reducing C . For a given beam $4QC^3 = (\omega_0/\omega)^2$ is approximately constant. Therefore, $4QC \sim 1/C^2$.

⁵ The use of a velocity jump following the helix for this purpose was suggested by A. Ashkin at the Sixteenth Annual Conf. on Electron Tube Res., Quebec, Can.; June 25-27, 1958.

⁶ J. R. Pierce, "Coupling of modes of propagation," *J. Appl. Phys.*, vol. 25, pp. 179-183; February, 1954. "The wave picture of microwave tubes," *Bell Sys. Tech. J.*, vol. 33, pp. 1343-1372; November, 1954.

mulated in this way, Haus and Robinson's⁷ theory of linear transducers is immediately applicable.

II. TRAVELING-WAVE INTERACTION IN TERMS OF MODE AMPLITUDES

In this section the theory of traveling-wave interaction is formulated in terms of the amplitudes of the circuit wave, slow space-charge wave, and fast space-charge wave. The usual small C approximation is made throughout in order to simplify the results. In terms of the physical variables: ac beam velocity, v_1 ; ac beam-charge density, ρ_1 ; ac beam-current density, i_1 ; ac circuit voltage, V_c ; and ac space-charge potential, V_{sc} ; the linearized equations of traveling-wave interaction are:

the electronic equation of motion,

$$j\omega v_1 + \frac{\partial}{\partial z}(u_0 v_1) = \frac{e}{m} \frac{\partial}{\partial z}(V_c + V_{sc}), \quad (1)$$

the equation of continuity,

$$j\omega \rho_1 + \frac{\partial i_1}{\partial z} = 0, \quad i_1 = \rho_0 v_1 + \rho_1 u_0, \quad (2)$$

the equation for the space charge voltage (from Poisson's equation)

$$\frac{\partial^2 V_{sc}}{\partial z^2} = -R^2 \frac{\rho_1}{\epsilon_0}, \quad (3)$$

$R = \text{space-charge reduction factor},$

and the circuit equation,

$$\left(\frac{\partial}{\partial z} + \Gamma_1\right) V_c = \pm j\beta_e \frac{K}{2} \sigma i_1, \quad (4)$$

where Γ_1 is the circuit propagation constant in the absence of the beam. K is the Pierce interaction impedance and σ is the cross-sectional area of the beam. The upper sign applies for forward-wave circuits and the lower sign applies for backward-wave circuits.

Introducing the mode variables defined in the manner suggested by Haus and Robinson,⁷

$$a_1 = \frac{V_c}{\sqrt{2K}} \text{ circuit mode amplitude} \quad (5)$$

$$a_2 = \frac{1}{2\sqrt{2W}} (V_1 - WI_1) \text{ slow space-charge mode amplitude} \quad (6)$$

$$a_3 = \frac{1}{2\sqrt{2W}} (V_1 + WI_1) \text{ fast space-charge mode amplitude} \quad (7)$$

where

$$W = 2 \frac{V_0}{I_0} \frac{\omega_p R}{\omega},$$

is the space-charge wave impedance of the electron beam, $V_1 = -(m/e)u_0 v_1$ is the kinetic voltage of the electron beam and $I_1 = \sigma i_1$ is the ac convection current in the electron beam. These variables have the property that their absolute square gives the power flow associated with that mode (except that $|a_2|^2$ gives the *negative* of the power flow associated with the slow space-charge mode). Upon substituting these new variables into (1)–(4), performing some straightforward algebraic manipulations, and making the small C approximation consistently, we obtain the equations for the mode amplitudes:

$$\left(\frac{\partial}{\partial z} + \Gamma_1\right) a_1 \pm j\kappa a_2 \mp j\kappa a_3 = 0 \quad (8)$$

$$-j\kappa a_1 + \left[\frac{\partial}{\partial z} + j(\beta_e + \beta_q)\right] a_2 = 0 \quad (9)$$

$$-j\kappa a_1 + \left[\frac{\partial}{\partial z} + j(\beta_e - \beta_q)\right] a_3 = 0, \quad (10)$$

where

$$\kappa = \frac{\beta_e}{2} \sqrt{\frac{K}{W}}$$

is the coupling constant between the circuit and fast and slow space-charge waves, $\beta_e = \omega/u_0$ is the electronic wave number, and $\beta_q = R\omega_p/u_0$ is the reduced plasma wave number. Eqs. (8)–(10) represent an extension of Pierce's coupling of modes of propagation theory⁶ to coupling between the *three* modes of a traveling-wave tube, together with an explicit expression for the coupling constant. Note that the circuit mode is coupled equally to the fast and slow space-charge waves⁸ and that the two space-charge waves are not coupled to each other.

It is convenient to extract a phase factor $e^{-j\beta_e z}$ from the definitions of the mode amplitudes by writing

$$a_i(z) = A_i(z) e^{-j\beta_e z} \quad (11)$$

and to express (8), (9), and (10) in terms of the dimensionless traveling-wave tube variables, b , d , $\xi = \beta_e C z$, $\beta_q/\beta_e C = \sqrt{4QC}$, and $\kappa/\beta_e C = k$:

$$\left[\frac{\partial}{\partial \xi} + jb \pm d\right] A_1 \pm jk A_2 \mp jk A_3 = 0 \quad (12)$$

$$-jk A_1 + \left[\frac{\partial}{\partial \xi} + j\sqrt{4QC}\right] A_2 = 0 \quad (13)$$

$$-jk A_1 + \left[\frac{\partial}{\partial \xi} - j\sqrt{4QC}\right] A_3 = 0. \quad (14)$$

It is of interest to note that the dimensionless coupling constant k is a function of QC only,

⁷ H. A. Haus and F. N. H. Robinson, "The minimum noise figure of microwave beam amplifiers," Proc. IRE, vol. 43, pp. 981–991; August, 1955.

⁸ The factor j preceding κ in (8)–(10) does not appear in Gould, *op. cit.* This difference is due to the choice of the *phases* of a_1 , a_2 , a_3 .

$$k^2 = \frac{1}{2\sqrt{4QC}} \quad (15)$$

or simply

$$A' = MA \quad (21)$$

To solve the three simultaneous first order linear equations, assume that each independent variable has a dependence on ξ of the form $e^{\delta\xi}$. For solutions of this type the determinant of the resulting algebraic equations must vanish

$$(\delta + jb \pm d)(\delta^2 + 4QC) \pm j = 0. \quad (16)$$

where M is a three by three-square matrix, and A' and A are three-element row and column matrices, respectively. A straight forward application of the solutions (17) to the case of initial conditions A_1 , A_2 , and A_3 yields the following expressions for the elements of the M matrix;

$$M_{11} = \sum_{i=1}^3 \frac{\delta_i^2 + 4QC}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \quad (22)$$

$$M_{21} = jk \sum_{i=1}^3 \frac{\delta_i - j\sqrt{4QC}}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \quad (23)$$

$$M_{31} = jk \sum_{i=1}^3 \frac{\delta_i + j\sqrt{4QC}}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \quad (24)$$

$$M_{22} = jk^2 \sum_{i=1}^3 \frac{(\delta_i - j\sqrt{4QC})(\delta_j + j\sqrt{4QC})(\delta_k + j\sqrt{4QC})}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \quad (25)$$

$$M_{32} = \pm k^2 \sum_{i=1}^3 \frac{1}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \quad (26)$$

$$M_{33} = -jk^2 \sum_{i=1}^3 \frac{(\delta_i + j\sqrt{4QC})(\delta_j - j\sqrt{4QC})(\delta_k - j\sqrt{4QC})}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \quad (27)$$

This is the familiar traveling-wave tube characteristic equation. A general solution may be written as the superposition of the three characteristic waves

$$A_i = \sum_{j=1}^3 C_{ij} e^{\delta_j \xi} \quad (17)$$

where certain relations exist between the C_{ij} by virtue of (12)–(14).

Let us apply these solutions to a length l of the traveling-wave section shown in Fig. 1 to find the mode amplitudes A_1' , A_2' , and A_3' at the output of the coupler when the input mode amplitudes are A_1 , A_2 , and A_3 .

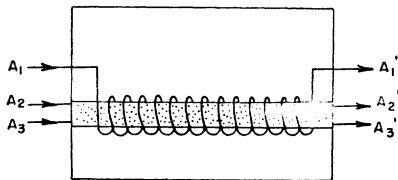


Fig. 1—Traveling-wave section.

We know in advance that the outputs are linearly related to the inputs

$$A_1' = M_{11}A_1 + M_{12}A_2 + M_{13}A_3 \quad (18)$$

$$A_2' = M_{21}A_1 + M_{22}A_2 + M_{23}A_3 \quad (19)$$

$$A_3' = M_{31}A_1 + M_{32}A_2 + M_{33}A_3, \quad (20)$$

where the subscripts i, j, k , are cyclical permutations of the integers 1, 2, 3, and $\xi = \beta_e Cl$. In writing (26) we have made use of the fact that

$$(\delta_i + j\sqrt{4QC})(\delta_j + j\sqrt{4QC})(\delta_k + j\sqrt{4QC}) = \mp j \quad (28)$$

a result which follows from the characteristic (16). We have not written the expressions for M_{12} , M_{13} , and M_{23} since it is possible to show, with the aid of (28), that

$$M_{12} = \mp M_{21} \quad (29)$$

$$M_{13} = \pm M_{31} \quad (30)$$

$$M_{23} = -M_{32}. \quad (31)$$

These expressions indicate certain symmetry properties of the coupler. For example, (29) states that a unit-amplitude slow wave at the input produces a circuit-wave amplitude at the output which is equal to (but of opposite sign in the case of the forward-wave amplifier) the slow-wave amplitude produced by a unit input to the circuit. Similarly, (30) states that a unit-amplitude fast-wave input produces a circuit-wave amplitude at the output which is equal to the fast-wave amplitude produced by a unit input to the circuit. Eq. (31) states that the fast wave produced by a unit slow-wave input is the negative of the slow wave produced by a unit fast-wave input.

In addition these matrix elements satisfy certain rela-

tions based on conservation of energy,⁷

$$M^+PM = P \quad (32)$$

$$MPM^+ = P, \quad (33)$$

where P is the parity matrix of Haus and Robinson

$$P = \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (34)$$

and M^+ is the Hermitian conjugate of M . In terms of the parity matrix the symmetry relations (29) to (31) can be written $(PMP)_{ij} = M_{ji}$ or $\widetilde{PMP} = M$ where the tilde indicates the transpose matrix.

To gain familiarity with the properties of the traveling-wave section and to answer the questions posed in the introduction, expressions for the matrix elements (22)–(27) have been evaluated using the Datatron 205 digital computer. A program now exists which evaluates each of the six matrix elements for specified QC , d , b , and ξ in about 30 seconds. The behavior for $QC=1$, $\frac{1}{4}$, $\frac{1}{8}$ ($d=0$) has been investigated by evaluating the matrix elements for $0 \leq \xi \leq 4.75$ and various values of b . Fig. 2 shows the magnitude of the matrix elements as a function of normalized length for the cases $QC=1$, $b=-2.0718$ (solid curve), and $QC=1$, $b=-1.5000$ (dashed curve). One may also think of $|M_{11}|$, $|M_{21}|$, and $|M_{31}|$ as the amplitudes of the circuit wave, slow and fast space-charge waves, as a function of distance along the coupler, which are produced when a unit amplitude is applied to the circuit. The Kompfner Dip condition ($|M_{11}|=0$) may be seen in the upper left. Some general features of large QC operation are seen in Fig. 2: coupling between the slow space-charge wave and either the circuit or the fast space-charge wave is small ($|M_{21}|$ and $|M_{31}|$ small), the slow space-charge wave goes through the coupler nearly unaffected ($|M_{22}| \cong 1$) at Kompfner Dip, and almost all of the fast-wave input is transferred to the circuit and very little remains on the beam.

The plot of $|M_{21}|$ in Fig. 2 suggests that there might be values of ξ and b for which M_{21} vanishes, resulting in no excitation of the slow space-charge wave for a circuit input. Additional calculations made to investigate this point indicate that such values of b and ξ probably do not exist, although M_{21} can be made small. It is, of course, possible to make M_{11} zero by proper choice of b and ξ , and this is the Kompfner Dip condition. When M_{11} is zero, the magnitude of M_{31} is always larger than the magnitude of M_{21} . This result follows directly from the fact that ac beam power at the output of the coupler must equal the circuit power at the input. When $|M_{31}|$ is greater than $|M_{21}|$ ⁹ it is possible, with the proper choice of a velocity jump, to completely remove the

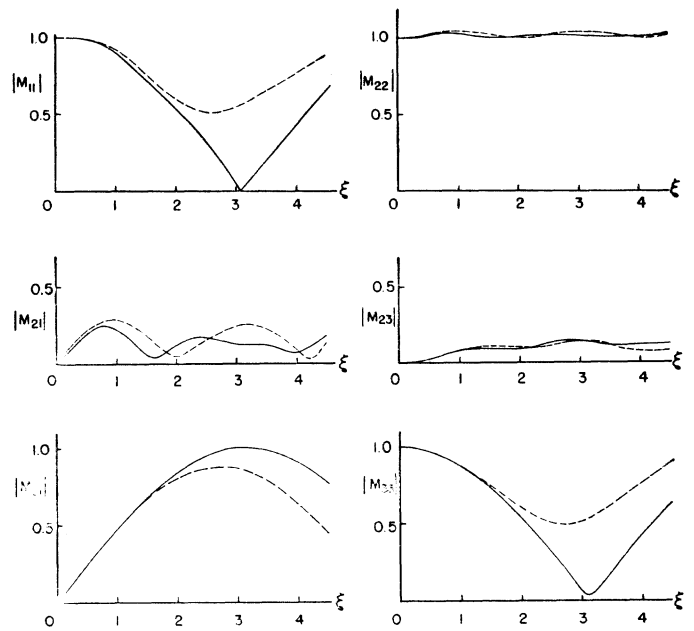


Fig. 2—Matrix elements for the case $QC=1$, $d=0$. Solid curves, $b=-2.0718$; dashed curves, $b=-1.5000$.

slow space-charge wave from the beam. This may be regarded as an impedance matching problem.¹⁰ It is also possible to make M_{33} equal to zero by the proper choice of b and ξ . The values of b and ξ which make M_{33} equal to zero are only slightly different from those which make M_{11} equal to zero.

An alternative method of evaluating the matrix elements which is more useful for rapid study of the properties of the coupler was also employed. Eqs. (12)–(14) are readily solved on an electronic differential analyzer. Since complex quantities are involved, each equation must be written in terms of real and imaginary parts, resulting in six first-order coupled differential equations. Voltages corresponding to the real and imaginary parts of one of the matrix elements are used as x and y inputs to an oscilloscope, giving a direct display of the matrix element in polar form with time as the independent variable. Fig. 3 shows such a display, the heavy portion of the trace representing the beginning of the coupler. The magnitude of M_{11} becomes very small at one point along the trace since b is very nearly equal to the value for Kompfner Dip. The constant b is varied by changing two potentiometer settings, hence the dip condition is readily found. Because the presentation of data is direct and rapid, this method is ideally suited for study of the matrix properties.

In a later section it is shown that matrix elements at the Kompfner Dip condition are of special interest. These have been computed for different QC values and the results are given in Table I. The way in which coupling to the slow space charge wave depends on QC may be seen by examining the column M_{21} .

⁹ $|M_{31}|$ will be larger than $|M_{21}|$ if $|M_{11}| < 1$, or if the traveling-wave section has no gain. This follows from the 1-1 component of (32).

¹⁰ S. Bloom and R. Peter, "A transmission line analog of a modulated electron beam," *RCA Rev.*, vol. 15, pp. 95–112; March, 1954.

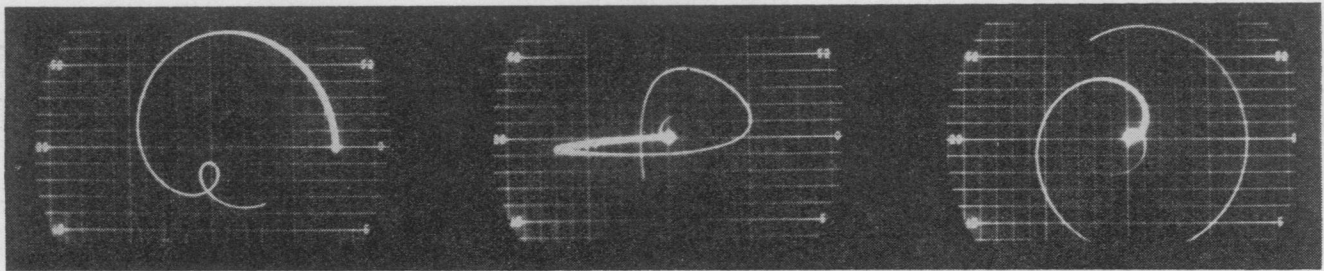


Fig. 3—Matrix elements for the case $QC=1$, $d=0$, $b=-2$, M_{11} (left), $-5jM_{21}^*$ (center), $-jM_{31}$ (right), $7\frac{1}{2}$ divisions = 1.0.

TABLE I
MATRIX ELEMENTS AT KOMPNER DIP ($M_{11}=0$)
(Magnitude and Phase Angle)

QC	b	ξ	M_{21}		M_{23}		M_{22}		M_{31}		M_{33}	
0.05	-1.5168	2.0037	1.0936	121.6°	1.6205	-131.3°	2.1959	-13.5°	1.4818	-176.2°	1.1959	-69.1°
0.10	-1.5121	2.0367	0.8339	108.0°	1.0859	-129.4°	1.6954	-39.3°	1.3021	-162.1°	0.6954	-39.6°
0.20	-1.5042	2.1129	0.5839	248.6°	0.6761	-124.6°	1.3409	-78.4°	1.1580	-139.8°	0.3409	9.1°
0.30	-1.5003	2.2502	0.3816	58.2°	0.4085	-113.8°	1.1456	-124.7°	1.0703	-111.0°	0.1456	77.1°
0.40	-1.5046	2.3325	0.3078	43.0°	0.3221	-105.4°	1.0947	-145.9°	1.0463	-96.5°	0.0947	115.1°
0.50	-1.5328	2.5070	0.2047	8.4°	0.2090	-79.9°	1.0419	176.9°	1.0207	-68.4°	0.0419	-156.7°
0.60	-1.6253	2.7315	0.1568	-49.0°	0.1587	-26.7°	1.0246	134.4°	1.0122	-30.0°	0.0246	-7.7°
0.80	-1.8918	2.9611	0.1696	-114.4°	0.1721	39.8°	1.0288	69.8°	1.0143	35.6°	0.0288	-170.2°
1.00	-2.0718	3.0871	0.1339	-156.1°	0.1351	78.2°	1.0179	17.7°	1.0089	84.4°	0.0179	-41.3°
1.20	-2.2333	3.2678	0.0983	139.7°	0.0883	139.2°	1.0097	-40.3°	1.0048	139.2°	0.0097	138.6°
1.40	-2.4152	3.4206	0.1016	79.9°	0.1021	-160.7°	1.0103	-95.1°	1.0051	-165.8°	0.0103	-46.4°
1.60	-2.5763	3.5131	0.0958	40.4°	0.0962	-121.8°	1.0092	-141.3°	1.0046	-120.1°	0.0092	7.8°
1.80	-2.7164	3.6131	0.0786	-3.9°	0.0788	-79.1°	1.0062	171.8°	1.0031	62.7°	0.0062	-150.0°
2.00	-2.8562	3.7327	0.0701	-60.7°	0.0703	-23.0°	1.0049	121.8°	1.0025	-25.5°	0.0049	12.2°

III. VELOCITY JUMPS, DRIFT SPACES, AND COMPOSITE SECTIONS

Since traveling-wave couplers will be used in conjunction with drift spaces and velocity jumps the matrices describing the latter are also presented here. In a drifting beam the phases of the fast and slow space-charge waves are delayed by $(\beta_e - \beta_q)l$ and $(\beta_e + \beta_q)l$ respectively if l is the drift distance. The amplitudes are unchanged. If we suppress the common phase delay $\beta_e l$ and define $\theta = \beta_q l$, the drift space equations are

$$A_2' = A_2 e^{-j\theta} \quad (35)$$

$$A_3' = A_3 e^{+j\theta}. \quad (36)$$

Although the circuit amplitude is not involved here it is convenient for matrix multiplication to use a three-by-three matrix and introduce the additional relation for the circuit amplitude $A_1' = A_1$. The matrix appropriate to a drift region is then

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j\theta} & 0 \\ 0 & 0 & e^{j\theta} \end{bmatrix} \quad \theta = \beta_q l. \quad (37)$$

The equations describing a velocity jump are obtained by noting that in a velocity jump the kinetic voltage V_1 and ac beam current I_1 are invariant in an abrupt jump.¹⁰ Using relations (6) and (7) the invariant principle is expressed

$$(A_2' + A_3')\sqrt{W'} = (A_2 + A_3)\sqrt{W} \quad (38)$$

$$(-A_2' + A_3')/\sqrt{W'} = (-A_2 + A_3)/\sqrt{W} \quad (39)$$

where the primed symbols refer to quantities after the jump and unprimed symbols refer to quantities before the jump. Solving for the matrix elements

$$M_{22} = M_{33} = \frac{1}{2} \left(\sqrt{\frac{W}{W'}} + \sqrt{\frac{W'}{W}} \right) \quad (40)$$

$$M_{23} = M_{32} = \frac{1}{2} \left(\sqrt{\frac{W}{W'}} - \sqrt{\frac{W'}{W}} \right). \quad (41)$$

To complete the matrix we again assume that $A_1' = A_1$, hence $M_{11} = 1$ and $M_{12} = M_{21} = M_{13} = M_{31} = 0$. Thus the matrix for a velocity jump is

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\alpha^2 + 1}{2\alpha} & \frac{\alpha^2 - 1}{2\alpha} \\ 0 & \frac{\alpha^2 - 1}{2\alpha} & \frac{\alpha^2 + 1}{2\alpha} \end{bmatrix} \quad \alpha = \sqrt{\frac{W}{W'}}. \quad (42)$$

It is of interest to note that the matrix for a jump from impedance W to impedance W' is the same as the matrix for a jump from W' to W except for a change in sign of the off-diagonal elements.

The matrix which describes a composite section consisting of cascaded individual sections of these types can

be written as the product of the matrices describing the individual sections. For example, a traveling-wave section (matrix M) followed by a drift region (matrix M') followed by a velocity jump (matrix M'') has the properties given by the resultant matrix

$$M'' M' M. \quad (43)$$

IV. FAST SPACE-CHARGE WAVE COUPLERS

We now apply the preceding results to synthesize fast wave couplers for longitudinal beam-type parametric amplifiers. First, consider the input coupler. The input coupler should perform two functions, 1) remove the noise from the fast space-charge wave, 2) place the input signal on the beam in the form of a fast space-charge wave as effectively as possible. If we describe the composite coupler by the matrix M (in general it will consist of a number of cascaded elementary sections), the first requirement can be stated

$$M_{32} = M_{33} = 0, \quad (44)$$

i.e. there should be no noise output on the fast space-charge wave due to noise inputs on *either* the fast or slow space-charge waves. Assume that such a coupler can be constructed and consider the restrictions imposed by the assumption that the coupler is lossless [(32) and (33)]. The 33 component of (33)

$$M_{31}M_{31}^* - M_{32}M_{32}^* + M_{33}M_{33}^* = 1 \quad (45)$$

together with (44) shows that

$$|M_{31}| = 1. \quad (46)$$

The 32 component of (33)

$$M_{31}M_{21}^* - M_{32}M_{22}^* + M_{33}M_{23}^* = 0 \quad (47)$$

together with (44) and (46) show that

$$M_{21} = 0. \quad (48)$$

The 31 component of (33)

$$M_{31}M_{11}^* - M_{32}M_{12}^* - M_{33}M_{13}^* = 0 \quad (49)$$

together with (44) and (46) show that

$$M_{11} = 0. \quad (50)$$

We conclude then that an input on the circuit must produce no output on the circuit and no output on the slow space-charge wave. The input signal is transferred completely to the fast space-charge wave. Thus the second requirement of the coupler is automatically satisfied. The remainder of the restrictions imposed by (32) and (33) are

$$|M_{22}|^2 - |M_{23}|^2 = 1 \quad (51)$$

$$-|M_{12}|^2 + |M_{13}|^2 = 1 \quad (52)$$

$$|M_{13}|^2 - |M_{23}|^2 = 1 \quad (53)$$

$$-|M_{12}|^2 + |M_{22}|^2 = 1 \quad (54)$$

$$M_{22}M_{12}^* = M_{23}M_{13}^* \quad (55)$$

$$M_{12}M_{13}^* = M_{23}^*M_{22}. \quad (56)$$

From (55) and (56), or from (51) and (53), or from (52) and (54), it is seen that

$$|M_{13}|^2 = |M_{23}|^2. \quad (57)$$

From (57) we may write

$$M_{13} = \gamma e^{-j\theta_3} \quad M_{23} = \gamma e^{-j\theta_2} \quad (58)$$

and the remaining relations (51) through (56) will be satisfied if

$$M_{12} = \sqrt{\gamma^2 - 1} e^{-j\theta_2} \quad M_{23} = \sqrt{\gamma^2 - 1} e^{-j\theta_3} \quad (59)$$

provided that $\gamma^2 \geq 1$. The resultant matrix is

$$M = \begin{pmatrix} 0 & \sqrt{\gamma^2 - 1} e^{-j\theta_2} & \gamma e^{-j\theta_3} \\ 0 & \gamma e^{-j\theta_2} & \sqrt{\gamma^2 - 1} e^{-j\theta_3} \\ e^{-j\theta_1} & 0 & 0 \end{pmatrix} \quad (60)$$

and there are four remaining variables γ , θ_1 , θ_2 , and θ_3 .

It has been shown that $M_{11} = M_{21} = 0$ is a necessary condition for the complete removal of beam noise from the fast space-charge wave. In a similar way it can be shown that if $M_{11} = M_{21} = 0$, then M_{32} and M_{33} are also zero. Thus the condition $M_{11} = M_{21} = 0$ is also sufficient.¹¹

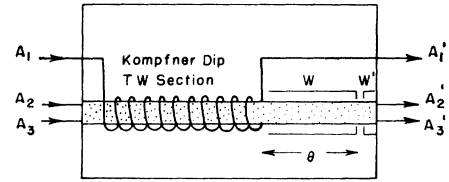


Fig. 4—Input coupler consisting of Kompfner Dip traveling-wave section, drift region, and velocity jump.

A coupler which has these properties can be constructed using a traveling-wave section in conjunction with velocity jumps and drift regions. Since M_{11} is to be zero the traveling-wave section *must* be operated at Kompfner Dip. Under this condition the fast-wave modulation of the beam is greater than the slow-wave modulation, hence it is possible to completely remove the slow wave with an appropriate velocity jump following the traveling-wave section. The physical configuration of the coupler is illustrated in Fig. 4. The matrix element connecting the circuit input to the slow-wave output of such a composite coupler is, using the results of the previous section,

$$M_{21} = \frac{\alpha^2 + 1}{2\alpha} e^{-j\theta} M_{21}' + \frac{\alpha^2 - 1}{2\alpha} e^{j\theta} M_{31}', \quad (61)$$

where $\alpha^2 = W/W'$ is the ratio of the beam impedance W' before the velocity jump to the beam impedance W

¹¹ It is of interest to note that these same general arguments also apply to lossless cavity couplers if one lets A_1 and A_1' refer to the incident and reflected waves, respectively, on the transmission line leading to the cavity system. For a cavity system to be considered as lossless, the energy dissipated in the cavity must be small compared to the power transferred to the beam.

after the jump, and M_{21}' and M_{31}' are matrix elements of the traveling-wave section alone. This may be made zero with either of two choices

$$\theta = \frac{1}{2} \arg \frac{M_{21}'}{M_{31}'} + n\pi, \quad \frac{W}{W'} = \frac{1 - \left| \frac{M_{21}'}{M_{31}'} \right|}{1 + \left| \frac{M_{21}'}{M_{31}'} \right|} \quad (62)$$

$$\theta = \frac{1}{2} \arg \frac{M_{21}'}{M_{31}'} + \left(n + \frac{1}{2}\right)\pi, \quad \frac{W}{W'} = \frac{1 + \left| \frac{M_{21}'}{M_{31}'} \right|}{1 - \left| \frac{M_{21}'}{M_{31}'} \right|}, \quad (63)$$

where θ is the length of the drift region, and W/W' is the ratio of the beam impedance before the jump to the beam impedance after the jump. The first choice corresponds to a jump to higher velocity and the second to a jump to lower velocity. The velocity jump locations for the two cases differ by a quarter space-charge wavelength. The magnitude and location of a jump to a higher velocity which makes $M_{21}=0$ is shown in Figs. 5 and 6. This coupler transfers the entire input signal to the fast space-charge wave and removes all beam noise from the fast space-charge wave. Its matrix has the same form as (60), where

$$\begin{aligned} \gamma &= |M_{13}'| = |M_{31}'| \\ \theta_3 &= \arg M_{13}' \\ \theta_2 &= \arg M_{12}', \end{aligned} \quad (64)$$

and M_{13}' and M_{12}' are elements of the traveling-wave matrix given in Table I. Furthermore by preceding the traveling-wave section by drift regions and velocity jumps it is possible to obtain other values of γ , θ_2 , and θ_3 without affecting the fundamental properties of the coupler expressed by (44), (48), and (50).

The requirements on the output coupler of a parametric amplifier are different from those of the input coupler. First, the output coupler should not be sensitive to a slow-wave input since the slow wave may be noisy (although the slow-wave noise will not be amplified if the pump is in the form of a pure fast wave), or

$$M_{12} = 0. \quad (65)$$

Furthermore, the coupling to the fast wave (M_{13}) should be maximized. The 11 component of (33) can be written

$$|M_{13}|^2 = 1 - |M_{11}|^2 + |M_{12}|^2. \quad (66)$$

From this relation it is seen that the coupling to the fast wave is maximized when

$$M_{11} = 0 \quad (67)$$

or when the traveling-wave section is operated at Kompfner Dip. When (65) and (67) are satisfied the energy conservation relations can be used to show that

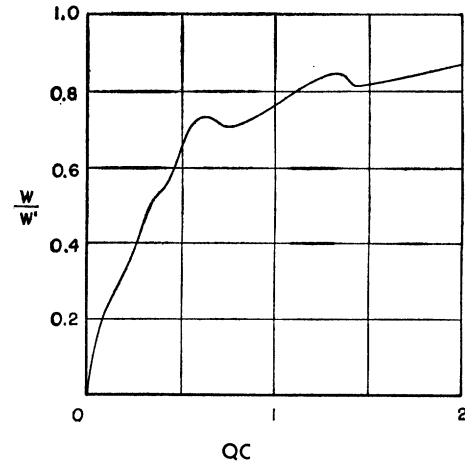


Fig. 5—Magnitude of impedance change required in velocity jump of the coupler illustrated in Fig. 5 in order for no coupling to the slow space-charge wave to occur ($M_{21}=0$).

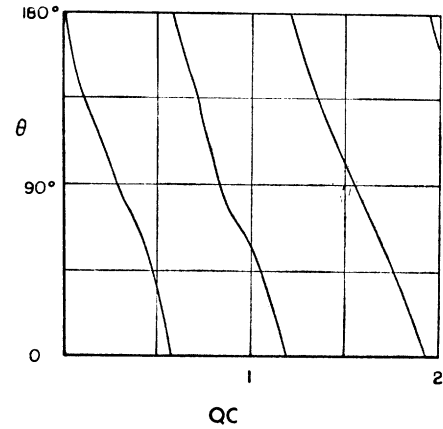


Fig. 6—Position of velocity jump of the coupler illustrated in Fig. 5 in order for no coupling to the slow space-charge wave to occur ($M_{21}=0$).

M_{23} and M_{33} are zero. It is possible to satisfy (65) by preceding the traveling-wave section by a velocity jump and drift region. The 12 matrix element of the composite coupler is

$$M_{12} = M_{12}' e^{-j\theta} \frac{\alpha^2 + 1}{2\alpha} + M_{13}' e^{j\theta} \frac{\alpha^2 - 1}{2\alpha}, \quad (68)$$

where $\alpha^2 = W'/W$ is the ratio of the beam impedance before the velocity jump to the beam impedance after the velocity jump,¹² and M_{12}' and M_{13}' are matrix elements of the traveling-wave section alone. This relation is similar to (61) which applies to the input coupler. By virtue of (29) and (30)

$$\frac{M_{12}'}{M_{13}'} = - \frac{M_{21}'}{M_{31}'} \quad (69)$$

It is possible to make M_{12} equal to zero in either of two ways:

¹² Note W' now refers to the beam impedance *before* the velocity jump and W refers to the beam impedance *after* the velocity jump. This is opposite from the convention of Section III.

$$\theta = \frac{1}{2} \arg \frac{M_{21}'}{M_{31}'} + n\pi \quad \frac{W}{W'} = \frac{1 - \left| \frac{M_{21}'}{M_{31}'} \right|}{1 + \left| \frac{M_{21}'}{M_{31}'} \right|} \quad (70)$$

or

$$\theta = \frac{1}{2} \arg \frac{M_{21}'}{M_{31}'} + \left(n + \frac{1}{2} \right) \pi \quad \frac{W}{W'} = \frac{1 + \left| \frac{M_{21}'}{M_{31}'} \right|}{1 - \left| \frac{M_{21}'}{M_{31}'} \right|} \quad (71)$$

Eqs. (70) and (71) are identical with (63) and (62). Thus the results shown in Figs. 5 and 6 are also applicable to the output coupler. In other words, the drift length which is required between the traveling-wave section and a velocity jump, in which the beam impedance is *increased* in order to make $M_{12}=0$, is the same as that required between the traveling-wave section and the velocity jump in which the beam impedance is decreased in order to make $M_{21}=0$. The resulting coupler is depicted in Fig. 7.

Similar arguments can be applied to synthesize a pump coupler, although the bandwidth afforded by a traveling-wave coupler is not required. The requirements for a pump coupler are 1) to produce no slow wave modulation ($M_{21}=0$), and 2) to maximize the fast-wave modulation (maximize M_{31}). The latter condition is achieved by taking $M_{11}=0$. Thus the pump coupler is electrically identical with the input coupler (although it operates at a different frequency).

Finally we consider the symmetric coupler shown in Fig. 8. The traveling-wave section is preceded by a velocity jump from impedance W' to impedance W and a drift region of length θ . It is followed by a drift region of length θ and a velocity jump back to the original impedance W' . It is readily verified that this composite coupler has the following symmetry

$$M_{12} = \mp M_{21}$$

$$M_{13} = \pm M_{31}$$

$$M_{23} = -M_{32}$$

which is the symmetry of the traveling-wave coupler alone. Furthermore by choosing the location and magnitude of the velocity jump in the manner already described (Figs. 5 and 6), it is possible to construct a kind of ideal coupler, whose matrix is

$$M = \begin{bmatrix} 0 & 0 & e^{-j\theta_3} \\ 0 & e^{-j\theta_2} & 0 \\ e^{-j\theta_1} & 0 & 0 \end{bmatrix}$$

The slow space-charge wave passes through the coupler with only a shift in phase, and there is a complete

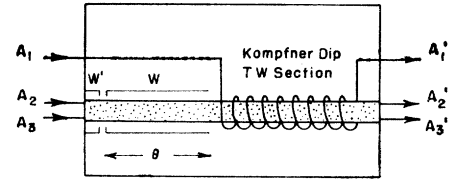


Fig. 7—Output coupler consisting of velocity jump, drift region, and Kompfner Dip traveling-wave section.

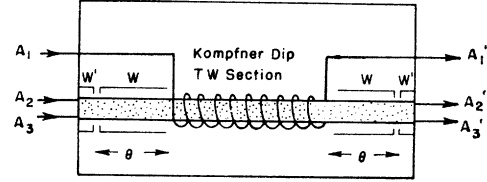


Fig. 8—Ideal coupler consisting of velocity jump, drift region, Kompfner Dip traveling-wave section, drift region, and velocity jump.

transfer of energy from the circuit wave to the fast space charge wave and vice versa.

V. DISCUSSION

The theory of longitudinal-beam traveling-wave couplers has been developed and applied to the design of couplers for parametric amplifiers. For any QC value a coupler can be constructed which couples only to the fast space-charge wave and, furthermore, this same coupler also removes the beam noise from the fast space-charge wave. This coupler consists of a traveling-wave section, drift region, and velocity jump. For certain QC values the velocity jump can be placed very close to the traveling-wave section, making a very compact coupler. Similarly, an output coupler which is sensitive only to the fast space-charge wave of the beam can be made by preceding a traveling-wave section with a velocity jump.

Results for the traveling-wave section by itself (Table I) indicate that the coupling to the slow space-charge wave is down 20 db or more at Kompfner Dip for $QC > 1.2$ ($|M_{21}| < .10$) and that beam noise is reduced by a similar amount ($|M_{32}| < .1$, $|M_{33}| < .01$) under these same conditions. When used with a low noise electron gun this less perfect but inherently simpler type of coupler also appears very attractive.

Finally, it should be pointed out that the presence of the pump signal on the beam may modify these results slightly. Locating the input coupler before the pump coupler will eliminate any possible effect in the input coupler where noise is eliminated from the fast wave.

VI. ACKNOWLEDGMENT

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